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Use of curved scanlines and boreholes to predict fracture frequencies

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Abstract

We advance the method of Hudson and Priest (Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 20 (1983) 73–89) to develop a method for a *curved* scanline to be used to predict the numbers of fractures that would be observed in any direction. When sampling along a scanline, the probability of intersecting a fracture is influenced by the relative orientations of the fracture and of the scanline at that location. This sampling bias can be minimised by the use of the Terzaghi correction, $w = (\cos\chi)^{-1}$, where χ is the angle between the scanline and the normal to the fracture. These corrected frequencies are used to simulate fracture frequencies for all other orientations by doubly-correcting the data. Modelled fracture frequency is contoured on a graph of simulated scanline plunge against simulated scanline azimuth. This method is based upon the assumption that the data collected along the scanline is representative of the fracture population when the Terzaghi correction has been applied.

A graph of cumulative frequency of fractures against distance along a scanline provides a simple method for determining whether the scanline crosses differently fractured areas. Frequencies are corrected for dip, strike, and both dip and strike, with data from homogeneously fractured areas plotting as straight lines. These frequencies can be normalised for ease of comparison.

Keywords: Fracture data; Densities; Scanlines; Measurements; Frequency

1. Introduction

1.1. The importance of scanlines

Sampling along a scanline is an important approach for collecting fracture information. In many situations it is either the easiest or the only way to collect data. For example, image log or core data from oil wells may be the only source of information available about fractures in reservoirs. Knowledge of fracture patterns and distributions is of great importance in hydrocarbon recovery, and so scanline data from borehole image logs need to be exploited to the fullest extent. For example, to enhance oil recovery within a fractured reservoir, drilling a well with an orientation to intersect the maximum number of fractures is typically desirable.

Scanlines are a quick and systematic collection technique for fracture data (LaPointe and Hudson, 1985). Yet the analysis will be complicated by the use of curved scanlines, or curved boreholes, and by the line crossing differently fractured areas.

1.2. Previous work using scanlines to predict fracture frequencies

Several approaches have been adopted to deal with sampling biases, particularly related to straight scanlines. Straight scanlines have been used to characterise fracture orientations and to make predictions about fracture frequencies, which is the reciprocal of fracture spacing along the line. Fracture sets are progressively under-sampled as the angle between the scanline and the fracture set decreases. Terzaghi (1965) introduced a factor w to correct this under-sampling of fracture frequencies from straight scanlines, namely:

$$w = (\cos\chi)^{-1}, \tag{1}$$

where χ is the angle between the scanline and the normal to a fracture. Eq. (1) is commonly applied to fracture data that are divided into sets, with the weighting factor calculated for the mean orientation of each set. Linear fracture

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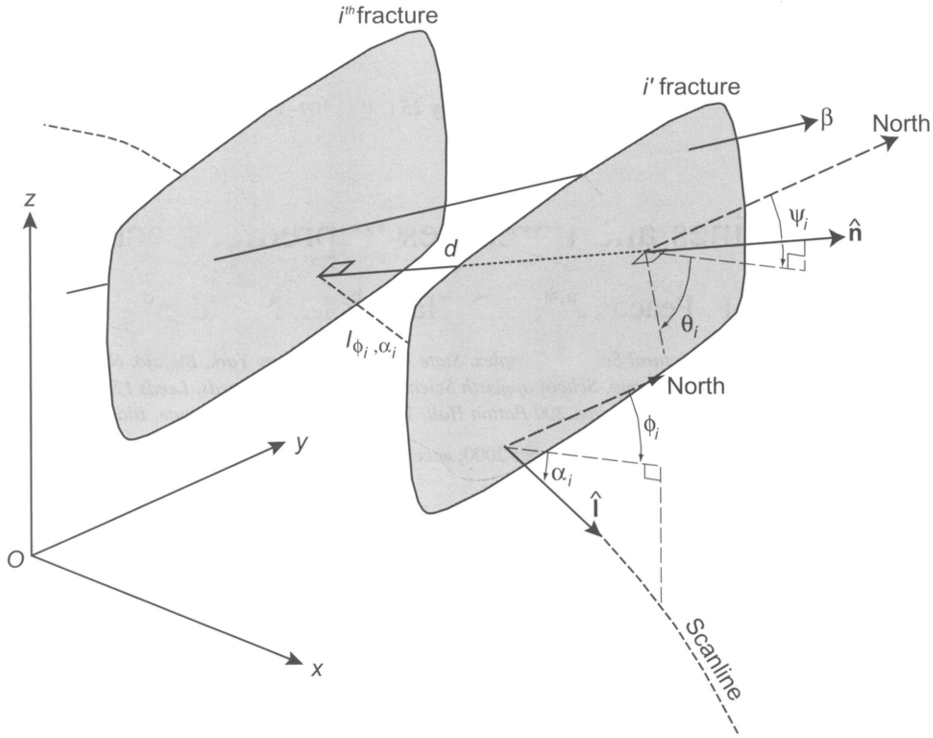


Fig. 1. Example of two parallel fractures transected by a curved scanline. Note that, to overcome the problem of fractures not forming several parallel sets, each fracture is assumed to belong to a separate set, i.e. the number of sets is the same as the number of fractures. See text for details.

frequency, which is the number of fractures observed or predicted to occur in a unit length, is the simplest and most commonly used measure of fracture frequency (Priest, 1993).

LaPointe and Hudson (1985, fig. 26a) used the Terzaghi correction to develop a *fundamental rosette* of corrected fracture frequencies, showing predicted frequencies as if all of the fractures on a plane on a plane. They verified the result using two sets of orthogonal scanlines to compare predicted and measured results. LaPointe and Hudson (1985, fig. 26b) also developed a rose diagram of fracture frequencies that would be encountered along straight scanlines in different directions. Their method is essentially two-dimensional, using strikes of fracture traces measured along a straight scanline.

Hudson and Priest (1983) and Priest (1993, chapter 4) used straight scanlines to predict fracture frequencies in all directions in three dimensions. Priest (1993, p. 112) presented a complex method to predict fracture maxima and minima, but stated that a more direct, but less elegant, method for determining fracture frequencies is to simply compute frequencies for the complete range of possible sampling directions. The latter approach is used here (Section 2). Priest (1993) did not discuss the use of curved scanlines. Lacazette (1991) expanded the method to model scanlines of all orientations, and showed how predicted fracture frequencies can be plotted on a stereogram.

Several approaches have been adopted to deal with sampling biases, particularly related to straight scanlines. Mauldon and Mauldon (1997) developed correction factors for sampling fractures along a borehole of non-zero radius, enabling predictions to be made about fracture frequencies from boreholes and tunnels. These factors require, however, prior knowledge of fracture size. Consequently, these correction factors are particularly useful when the borehole radius is large in comparison with the length of fracture intersects, which is the situation in which fracture size may be better approximated. Their approach reduces to the Terzaghi correction factor for a borehole of zero radius, i.e. a scanline. Martel (1999) analysed fracture orientation data from boreholes using the mean orientation of fractures, spherical variance and the moment of inertia to analyse fracture pole orientations distributed on a hemisphere. The method considers the effect of borehole sampling bias on measured orientations of fractures with a pre-assumed orientation distribution, such as uniform, and then modifies the distribution using the mismatch between observations and predictions for a particular case. The significance of the mismatch can be evaluated visually on a stereogram, or quantitatively with chi-square or Kolmogorov–Smirnov tests (e.g. Davis, 1986; Martel, 1999).

Grossenbacher et al. (1997) present a method for determining fracture frequencies from data collected along circular scanlines, which they suggest can be expanded and adapted to the more general case of irregularly curved scanlines. Mauldon et al. (1999, 2001) use circular scanlines to avoid sampling biases. They also develop methods for quantifying the intensity, density and mean trace-lengths of fracture traces within the circle. This method is useful when fracture traces can be measured on surfaces.

All of these approaches offer improvements in resolving bias issues, but none exploit the strategy proposed in this paper of considering fracture frequency along curvilinear lines.

1.3. Aims of this paper and simplifying assumptions

This paper describes a method that uses data collected along a curved scanline to estimate the numbers of fractures that would be intersected along straight scanlines of any orientation through the same statistically homogeneous rock mass (Section 2). In this paper, the term *fracture* is used to describe any brittle planar discontinuity in rock, including faults, joints, veins and dykes. The method identifies the scanline orientations that intersect the maximum and minimum numbers of fractures. The method is a more generalised version of the methods of LaPointe and Hudson (1985, fig. 26b) and Priest (1993, chapter 4) for predicting spatial distributions of fracture frequencies. Both of these methods require straight scanlines and the first only considers fracture strikes. By contrast, we give a mathematical formulation for the three-dimensional case where the dip and dip direction of fractures are recorded along straight or curved scanlines. This method is particularly useful for the analysis of borehole data, since boreholes are commonly curved.

We also present a graphical representation of the cumulative number of fractures versus distance along the scanline as a simple way to identify fracture domains with different frequencies and patterns (Section 3). This representation tests the assumption that the fracture distribution is spatially uniform.

2. Method to predict fracture frequencies using scanlines

2.1. Data collection

A scanline can be produced by placing a tape measure straight across an exposure and measuring the fractures that intersect the tape measure. Similar data can be obtained from borehole image data or core from a well. Characteristics that may be measured along a scanline or borehole include (Fig. 1): the local plunge and azimuth of the scanline or borehole at each fracture location; the dip and dip direction of each fracture; the distance traversed along the scanline or borehole between adjacent fractures; the fracture type, e.g. fault, joint, vein; and other data, such as any displacements, mineralogy or relative ages.

2.2. Simple case of a straight horizontal scanline intersecting vertical fractures

Take the simple case of a straight horizontal scanline that intersects a number (N) of vertical fractures. As the angle between the fracture strike and the scanline decreases, the probability of the fracture being sampled by the scanline decreases. At the location of the ith fracture along the scanline, the local azimuth of the scanline segment is ϕ_i. Let the azimuth of the normal to the ith fracture trace be ψ_i. If the frequency, f_{ϕ_i}, of similarly striking fractures is known along the scanline segment with azimuth ϕ_i, then the frequency of fractures, f_β, along any other scanline with a constant azimuth β can be predicted.

Suppose that the fracture observed along the scanline is separated from a similarly striking fracture by a distance l_{ϕ_i} along the scanline at the location of the ith fracture. Then, the perpendicular separation of these same two fractures is:

$$d = l_{\phi_i}|\cos(\phi_i - \psi_i)| \tag{2}$$

and the distance l_β between these two fractures along a scanline with azimuth β is equal to:

$$l_\beta = \frac{d}{|\cos(\beta - \psi_i)|} = \frac{|\cos(\phi_i - \psi_i)|}{|\cos(\beta - \psi_i)|} l_{\phi_i}. \tag{3}$$

The frequency, f_β, of fractures with orientation ψ_i along a scanline with azimuth β is then:

$$f_\beta = \frac{1}{l_\beta} = \frac{|\cos(\beta - \psi_i)|}{|\cos(\phi_i - \psi_i)|} f_{\phi_i}. \tag{4}$$

If we define L to be the length of the scanline, we can use L as a normalising factor for determining fracture frequencies. The use of fracture frequencies per unit length as opposed to absolute fracture numbers enables comparisons between scanlines with different lengths and orientations. If the scanline is straight, as in this case, then the frequency of fractures along the scanline, N/L, is the number of fractures per metre. From Eq. (4), we can estimate the fracture frequency per unit length along a scanline with azimuth β to be:

$$f_\beta = \frac{1}{L} \sum_{i=1}^{N} \frac{|\cos(\beta - \psi_i)|}{|\cos(\phi_i - \psi_i)|}, \tag{5}$$

based upon a summation over all of the observed N fractures.

To determine the directions in which the maximum and minimum numbers of fractures would be intersected, we simulate the full range of straight scanline azimuths $0° \le \beta \le 180°$ using Eq. (5).

2.3. Curved scanline intersecting fracture planes

Section 2.2 describes the simple case of a scanline in two-dimensional space. Now consider the more general case of a scanline in three-dimensional space that intersects a number (N) of fractures. The discussion below makes reference to a pair of parallel fractures (Fig. 1). At the end of Section 2.3, however, this requirement is dropped such that the method is applicable to any system of fractures intersected by a

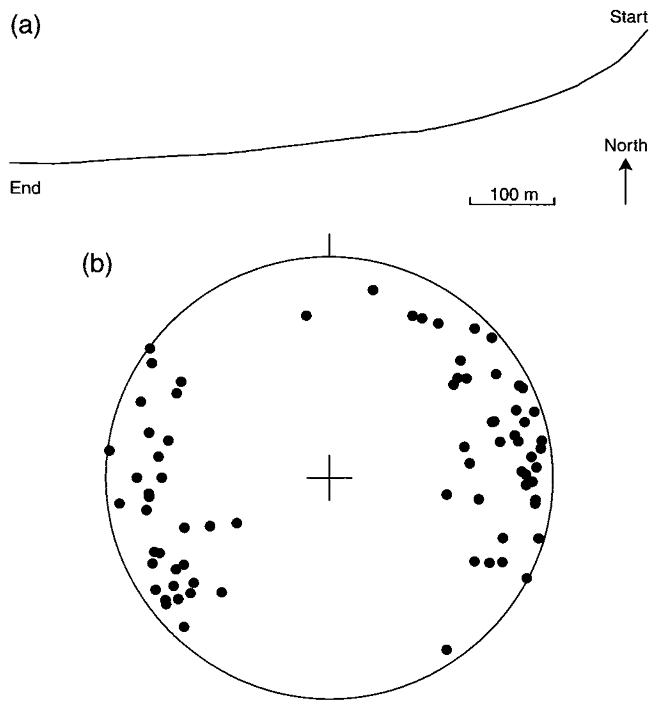


Fig. 2. (a) Map of the borehole described in Section 2.4, showing change in orientation along its length (from azimuth 214° to azimuth 272°). (b) Lower hemisphere equal area stereogram of poles to fractures intersected by the borehole ($n = 76$).

scanline, regardless of whether they have any orientations in common.

At the location of the ith fracture along the scanline, the local azimuth of the scanline segment is ϕ_i and the plunge is α_i (Fig. 1). The observed fracture is described with reference to its dip angle θ_i and its dip direction ψ_i. If the frequency of fractures, f_{ϕ_i,α_i}, with the same dip and dip direction is known along the direction of a straight segment of the scanline with azimuth ϕ_i and plunge α_i, then the frequency of fractures, $f_{\beta,\gamma}$, along any other scanline with a constant azimuth β and a constant plunge γ can be predicted using the following argument.

Suppose that a fracture (i) observed along the scanline is separated from a parallel fracture (i') (which may or may not be observed) by a distance l_{ϕ_i,α_i} along a tangent drawn to the scanline at the location of the ith fracture (Fig. 1). Then l_{ϕ_i,α_i} denotes the straight-line distance between the points where this tangent line intersects the planes. With reference to a Cartesian (x,y,z) co-ordinate system, where y is north and the z-axis is vertically upward, the unit vector along the direction of the scanline segment at the location of the ith fracture is:

$$\hat{\mathbf{l}}_{\phi_i,\alpha_i} = (\sin\phi_i\cos\alpha_i, \cos\phi_i\cos\alpha_i, -\sin\alpha_i) \quad (6)$$

and the *upward* unit normal vector to the fracture in the dip direction is:

$$\hat{\mathbf{n}} = (\sin\psi_i\sin\theta_i, \cos\psi_i\sin\theta_i, \cos\theta_i). \quad (7)$$

Since $|\hat{\mathbf{l}}_{\phi_i,\alpha_i}\cdot\hat{\mathbf{n}}|$ defines the cosine of the angle between the scanline segment and the normal to the ith fracture, the perpendicular separation of fractures i and i' is:

$$d = l_{\phi_i,\alpha_i}|\hat{\mathbf{l}}_{\phi_i,\alpha_i}\cdot\hat{\mathbf{n}}| = l_{\phi_i,\alpha_i}|\cos\alpha_i\sin\theta_i\cos(\phi_i - \psi_i) - \sin\alpha_i\cos\theta_i|. \quad (8)$$

The average distance, $l_{\beta,\gamma}$, between fractures of the ith fracture set along a line in the direction $\hat{\mathbf{l}}_{\beta,\gamma}$, with azimuth β and plunge γ, is:

$$l_{\beta,\gamma} = \frac{d}{|\hat{\mathbf{l}}_{\beta,\gamma}\cdot\hat{\mathbf{n}}|} = \frac{|\hat{\mathbf{l}}_{\phi,\alpha}\cdot\hat{\mathbf{n}}|}{|\hat{\mathbf{l}}_{\beta,\gamma}\cdot\hat{\mathbf{n}}|}l_{\phi_i,\alpha_i}. \quad (9)$$

The frequency, $f_{\beta,\gamma}$, of the ith fracture set in the direction $\hat{\mathbf{l}}_{\beta,\gamma}$ is then:

$$f_{\beta,\gamma} = \frac{1}{l_{\beta,\gamma}} = \frac{|\cos\gamma\sin\theta_i\cos(\beta - \psi_i) - \sin\gamma\cos\theta_i|}{|\cos\alpha_i\sin\theta_i\cos(\phi_i - \psi_i) - \sin\alpha_i\cos\theta_i|}f_{\phi_i,\alpha_i}, \quad (10)$$

by expanding the dot products in Eq. (9). Defining L to be the length of the measured scanline, the predicted fracture frequency for the entire line will be:

$$f_{\beta,\gamma} = \frac{1}{L}\sum_{i=1}^{N}\frac{|\cos\gamma\sin\theta_i\cos(\beta - \psi_i) - \sin\gamma\cos\theta_i|}{|\cos\alpha_i\sin\theta_i\cos(\phi_i - \psi_i) - \sin\alpha_i\cos\theta_i|}, \quad (11)$$

by summing the contributions from each individual fracture. To determine the scanline directions that would intersect the maximum and minimum number of fractures, we simulate the full range of scanline azimuths $0° \leq \beta \leq 360°$ in combination with the full range of plunge angles $0° \leq \gamma \leq 90°$ (Priest, 1993, p. 112). Note that, to overcome the problem of fractures not forming several parallel sets, each fracture is assumed to belong to a separate set, i.e. the number of sets is the same as the number of fractures. The Terzaghi correction is applied to each fracture individually (also see Priest, 1993, p. 110).

An alternative mathematical justification of the construction leading to Eq. (11) is presented in Appendix A.

2.4. Example of a curved borehole

We illustrate our method using fracture data detected with borehole imagery from a subhorizontal well in a fractured carbonate reservoir. Fig. 2a shows a map view of the borehole, illustrating its curvature. The data are proprietary, and so the location, depth and stratigraphic host may not be identified. The well intersects 76 mostly steeply dipping fractures over a distance of approximately 803 m. These fractures are mostly open joints with apertures of up to a few millimetres. A stereogram of the poles to the fractures is shown in Fig. 2b. The well has an azimuth between 214° and 272°, and so curves by almost 60°.

Predicted fracture frequencies for other sampling directions were determined by the application of Eq. (11) with $N = 76$ and $L = 803$ m. A simple computer program was

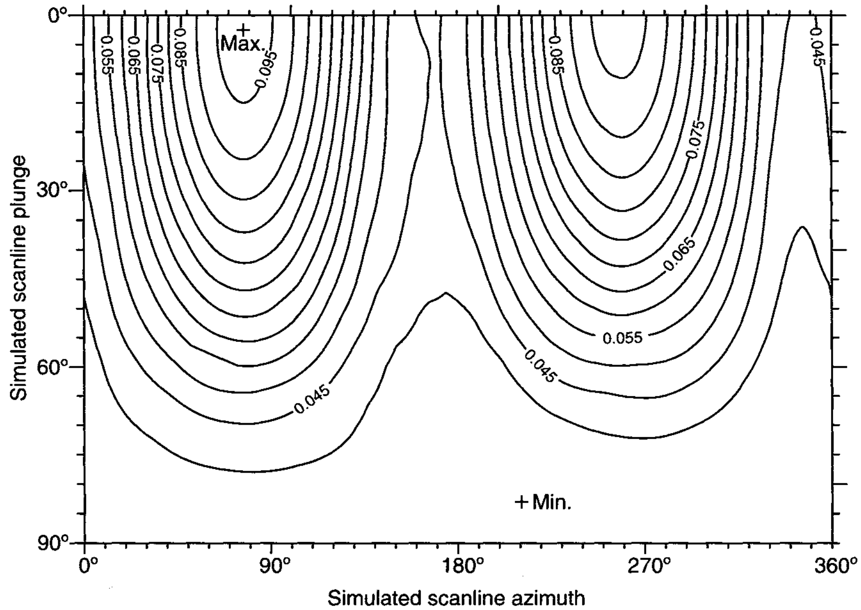


Fig. 3. Contours of the predicted fracture frequency (m^{-1}) as a function of simulated scanline plunge and simulated scanline azimuth. The maximum number of fractures would be intersected along a line plunging at about 3° towards 077°, whilst the minimum number of fractures would be intersected along a line plunging at about 83° towards 210°.

written to simulate fracture frequencies in all orientations, using 0.5° bins for simulated scanline azimuths and dips. The predicted fracture frequencies for the simulated scanlines are shown in Fig. 3, these being contoured using a standard contouring computer program. In this particular case, fractures are detrimental to production, and so the prediction of the orientation for the minimum number of intersected fractures was required.

2.5. Simple two-dimensional test of the method

Fig. 4 illustrates a simple two-dimensional test of the method presented in Sections 2.2 and 2.3. A schematic map with two sets of evenly spaced fractures has been produced, with two straight and two curved scanlines superposed on the fracture map (Fig. 4a). The method presented in Section 2.3 has been used to estimate the numbers of fractures in other orientations. The results shown in Fig. 4b demonstrate the accuracy of the technique.

2.6. Sampling techniques and limitations of the method

The method presented in Section 2.3 can be used to estimate fracture frequencies along a scanline from data collected along a different straight scanline. Larger sample sizes improve the quality of estimates of fracture frequency (e.g. Peacock and Sanderson, 1993). Tens or hundreds of measurements are probably needed to give meaningful results. Many practical situations restrict sampling to particular lines, e.g. boreholes, mines and tunnels. Where sampling is less restricted, it may be possible to combine data from different traverses of different orientation to enhance the accuracy of the frequency description. Average fracture frequencies obtained along a scanline only reasonably represent the fracture system if the sample is within a statistically homogeneous region. If fracture frequency is not homogeneous along a scanline, then the sample should be subdivided into homogeneous sub-domains (Wojtal, 1989). This issue is addressed in the next section.

3. Test of the homogeneity of fracture data collected along a scanline

3.1. Example of the borehole

A graph of cumulative number of observed fractures against the distance traversed along a scanline (Fig. 5a) may be plotted to test whether the data obtained along the scanline are from a single homogeneous fracture pattern. To test whether artefacts affect this graph, the Terzaghi correction is applied to each observed fracture. The Terzaghi correction may be applied to: (1) the angle between the scanline and the dip of each fracture, (2) the angle between the scanline and the strike of each fracture, or to (3) the angle between the scanline and the fracture. Data that plot as a straight line indicate that fracture frequency is constant along the scanline. Gradient changes indicate variations in fracture frequency, implying that the scanline data set should be subdivided by separating the dataset at the changes of slope. Differences in the slopes of the graphs for the uncorrected data and the data corrected using the Terzaghi factor imply that the sampling has not been truly representative. These graphs should be plotted to test whether the scanline cuts across a domain of homogeneous

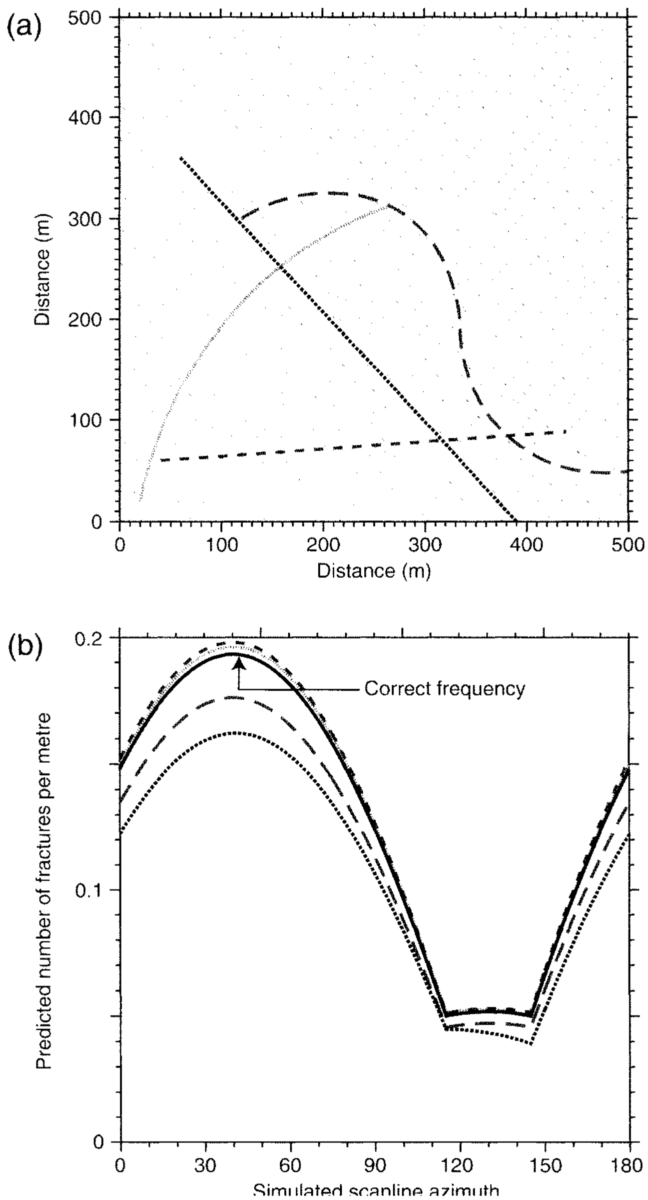


Fig. 4. Test of the method presented in Section 2.3. (a) Map of two fracture sets, with two straight scanlines and two curved scanlines. (b) Test results, showing actual frequencies (solid) as a function of scanline azimuth, and predicted frequencies (dashed curves) determined using the method presented in Section 2 for the fracture map and scanlines shown in (a).

fracture frequency before commencing the type of analysis described in Section 2.

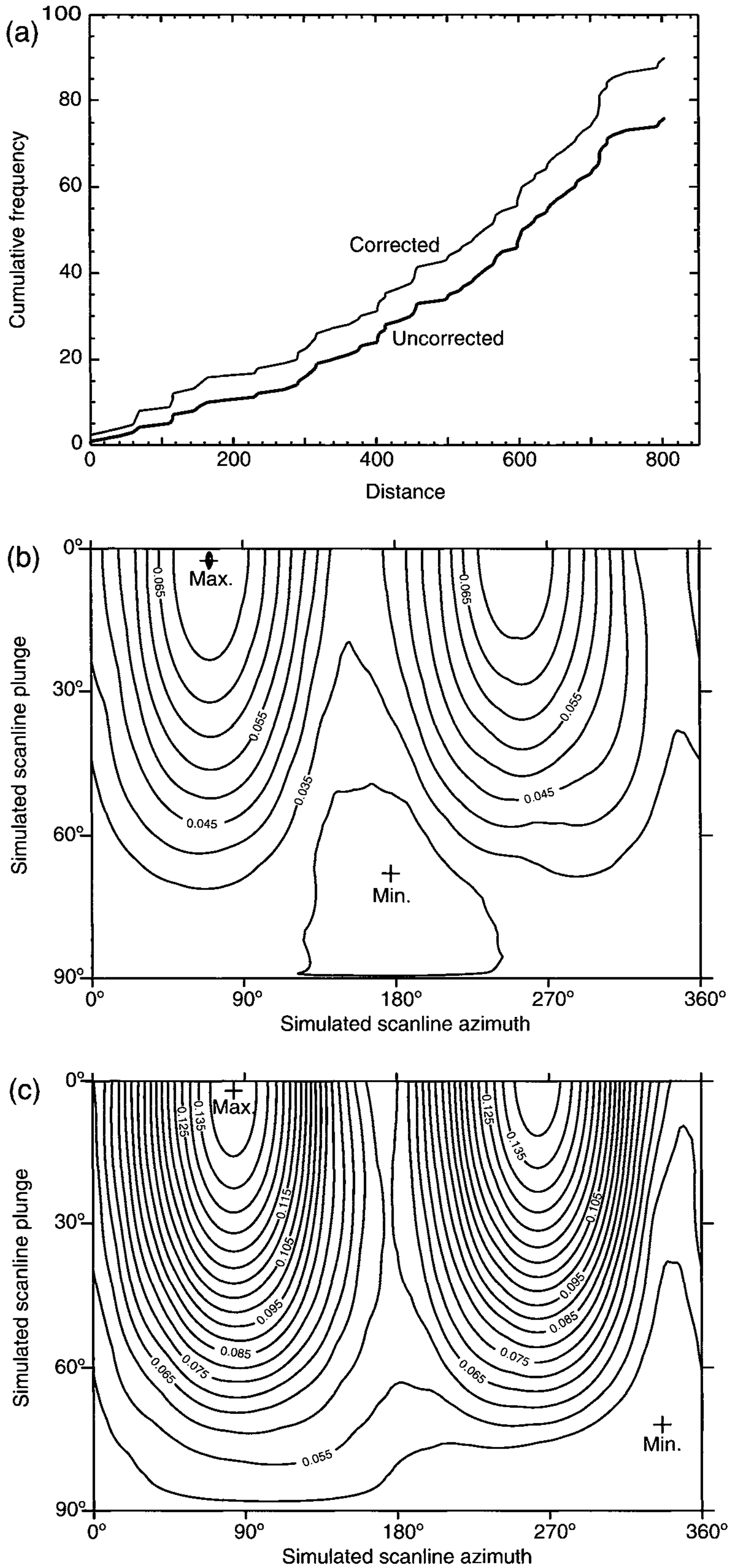
The data from the borehole described in Section 2.4 show a curve in the graph of cumulative number of observed fractures against the distance traversed along a borehole (Fig. 5a). The slope steepens (indicating increased fracture frequency) towards the end of the borehole. When the data are divided, the two parts of the borehole produce similar contours and predictions of the orientations of scanlines to intersect the maximum numbers of fractures (compare Fig. 5b and c). The predicted values are, however, different. This pattern is interpreted to be caused by changes in fracture frequency along the scanline rather than by changes in fracture orientations.

3.2. Example of faults at Flamborough Head, Yorkshire

Although emphasis has been placed on borehole data, the methods presented in this paper can also be used for any curved scanline, including field data. The normal faults that are exposed in the Cretaceous Chalk cliffs on the south coast of Flamborough Head, East Yorkshire, have displacements of up to 6 m and represent about 1% extension in all horizontal directions (Peacock and Sanderson, 1993, 1994). 1339 faults have been recorded along a 6058 m scanline, yielding an average frequency of 0.221 fractures per metre. The scanline consists of three approximately straight and horizontal portions, orientated toward 071°, 088° and 063° (Peacock and Sanderson, 1993, table 1). These data are used because Flamborough Head provides a good example of fracture orientations measured along a curved scanline. Fault displacements are not considered.

Fig. 6a shows a graph of the cumulative number of faults against distance along the scanline at Flamborough Head. For ease of comparison, the uncorrected and corrected values can be normalised and re-plotted, as shown in Fig. 6b. This normalisation is carried out by determining the cumulative of the uncorrected or corrected values, and dividing by the final sum of these values. The slopes become shallower after a distance of about 4180 m (at around fault number 1040), and so the data set has been divided at this location for the analysis (Fig. 7). The two parts of the scanline again produce similar predictions of the orientations of scanlines to intersect the maximum and minimum numbers of faults (compare Fig. 7a and b). The predicted frequencies are, however, different. Again, this pattern is interpreted to be caused by changes in fault frequency along the scanline rather than by changes in fault orientations. This approach is therefore of use in comparing areas with different fracturing characteristics.

Fig. 5. (a) Graph of the cumulative number of fractures, and of corrected cumulative number of fractures, against distance along the borehole described in Section 2.4. The *corrected* graph has been corrected for the angle between the borehole and the strike of the fracture using the Terzaghi (1965) correction. There is a change in slope along the scanline, with higher fracture frequency towards the end. (b) Contours of predicted fracture frequency (m^{-1}) as a function of simulated scanline plunge and simulated scanline azimuth for the first 35 fractures. The maximum number of fractures (0.075 m^{-1}) would be intersected along a scanline plunging at 2.5° towards 070°, while the minimum number of fractures (0.027 m^{-1}) would be intersected along a scanline plunging at 68° towards 177°. (c) Contours of predicted fracture frequency (m^{-1}) as a function of simulated scanline plunge and simulated scanline azimuth for fractures 36–76. The maximum number of fractures (0.144 m^{-1}) would be intersected along a scanline plunging at 2° towards 083.5°, while the minimum number of fractures (0.045 m^{-1}) would be intersected along a scanline plunging at 72° towards 336.5°.


(a)
Cumulative frequency
Corrected
Uncorrected
Distance
(b)
Max.
Min.
Simulated scanline plunge
Simulated scanline azimuth
(c)
Max.
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Simulated scanline plunge
Simulated scanline azimuth

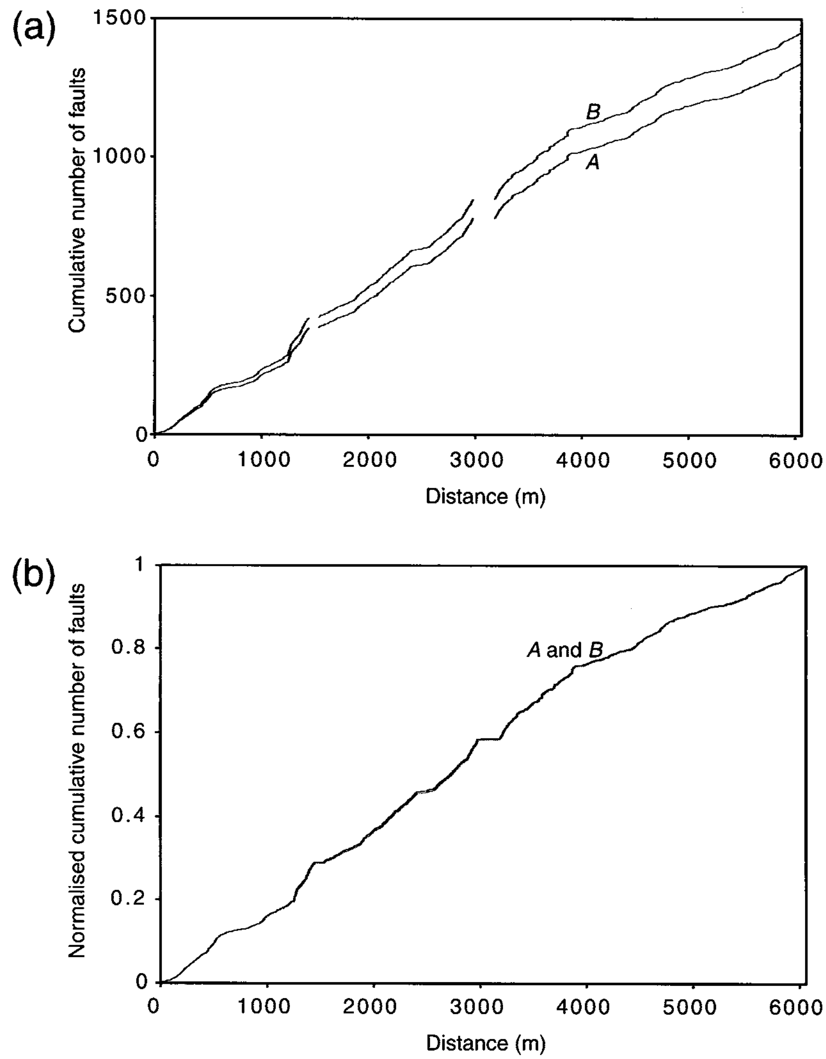

Fig. 6. Data for 1339 normal faults from the Cretaceous Chalk at Flamborough Head, East Yorkshire, England (Peacock and Sanderson, 1993, 1994). (a) Cumulative number of faults against distance along scanline: A = measured, B = Terzaghi-corrected. There is a change in slope of the curves at about 4180 m. (b) The same data as (a) but normalised by the cumulative number of fractures. The data set includes three short sample breaks. Straight lines indicate a homogeneously fractured area.

4. Conclusions

We have formulated an approach that enables the orientations of fractures obtained along a single straight or curved scanline to be used to estimate the numbers of fractures in any other direction. When sampling along a line, the probability of intersecting a fracture is affected by the relative orientations of the scanline and the fracture. This sampling bias can be lessened by the use of the Terzaghi correction, which must be applied to each measured fracture individually. The corrected frequencies of all of the fractures measured along the scanline are then used to simulate fracture frequencies in all other orientations, by doubly-correcting the data. The correction is applied to each fracture individually, using the angle between the fracture and the scanline at that point. The modelled fracture frequency may be contoured as a function of the simulated line plunge and the simulated line azimuth, and predictions of the orientations of the fracture frequency extrema can be made. This method is a further development of that presented by LaPointe and Hudson (1985) and Priest (1993).

The method to predict fracture frequencies is based upon the assumption that the Terzaghi-corrected data collected along the scanline are representative of the fracture population. The homogeneity of the measured data can be tested using a graph of cumulative frequency of fractures against distance along the scanline. To test for sampling biases caused by under-sampling of fractures at a low angle to the scanline, the cumulative frequency can be corrected using the Terzaghi factor (Eq. (1)) for fracture dip, dip direction and both, and plotted against distance along the scanline. Uncorrected and corrected frequencies can be normalised for ease of comparison, with straight lines indicating homogeneously fractured areas. Straightness of

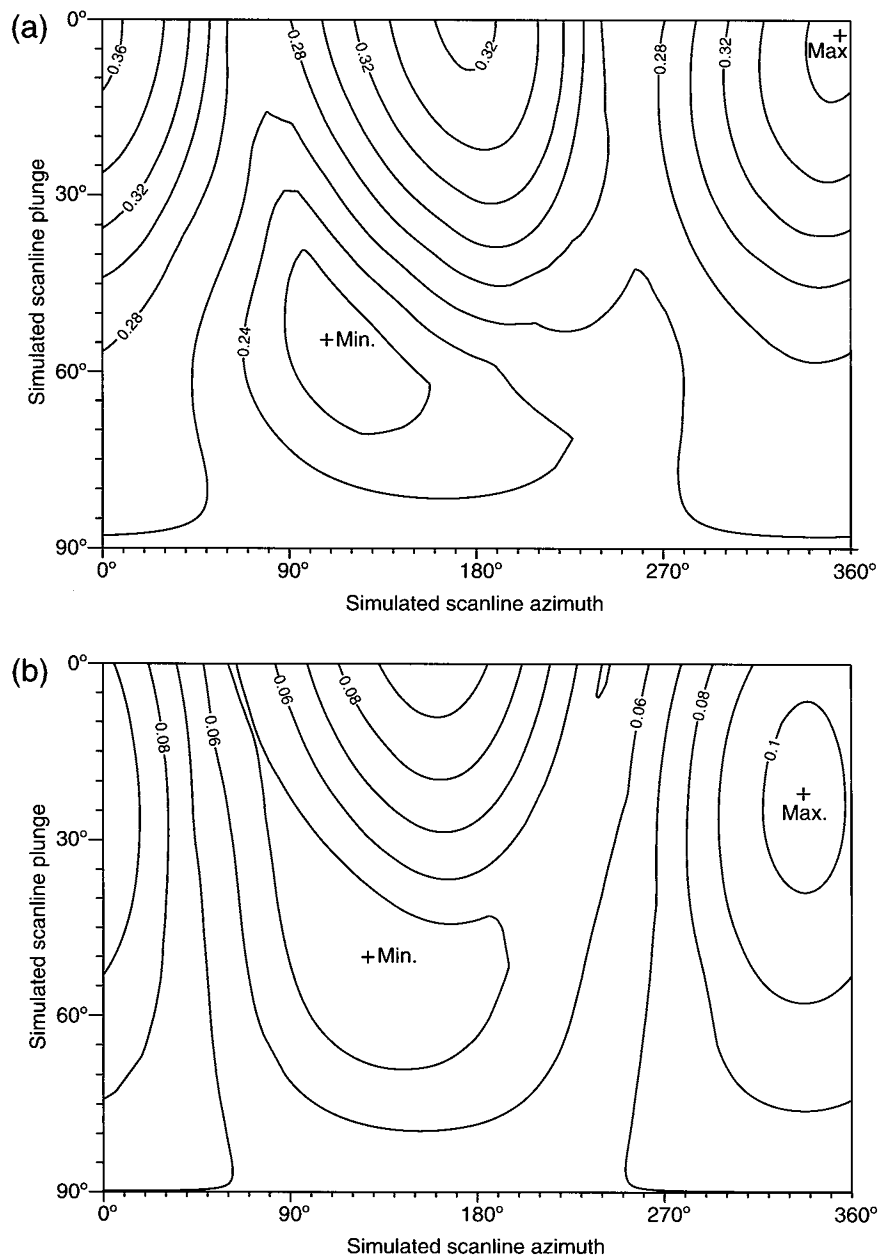

Fig. 7. Contours of predicted fracture frequency (m^{-1}) as a function of simulated scanline plunge and simulated scanline azimuth. (a) Graph for the 1040 faults over the first 4180 m of scanline at Flamborough Head. The maximum number of faults (0.37 m^{-1}) would be intersected along a scanline plunging at 3° towards 355°, while the minimum number of faults (0.2 m^{-1}) would be intersected along a scanline plunging at 54° towards 108°. (b) Graph for the 299 faults over the last 1878 m of scanline. The maximum number of faults (0.1 m^{-1}) would be intersected along a scanline plunging at 22° towards 337°, while the minimum number of faults (0.04 m^{-1}) would be intersected along a scanline plunging at 50° towards 127°.

the graph can be tested visually or by standard statistical techniques.

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Appendix A

For convenience, we refer to the curved scanline along which measurements were made as scanline Alpha, and to a second straight scanline as scanline Beta. We first use data collected along scanline Alpha to determine the frequency of the fracture set j on scanline Beta. We then extend the argument to multiple fracture sets, or to a collection of individual fractures with no orientations in common. The rock mass is assumed to contain n fracture sets. We divide scanline Alpha into m straight-line segments, where m can be any number and the segments can be arbitrarily short in length. We define the following:

$L(\alpha)$ = total length of scanline Alpha,
$L_k(\alpha)$ = length of the kth segment of scanline Alpha,
$\hat{\mathbf{n}}_j$ = unit normal for the jth fracture set,
$\hat{\boldsymbol{\beta}}$ = unit vector in the direction of scanline Beta,
$\hat{\boldsymbol{\alpha}}_k^s$ = unit vector in the direction of the kth *segment* of scanline Alpha,
$\hat{\boldsymbol{\alpha}}_i^f$ = unit vector in the direction of scanline Alpha at the location of the ith *fracture*,
$N_{j,k}$ = number of fractures of set j crossed by the kth segment of scanline Alpha,
f_j = true frequency of the jth fracture set, and
$f'_j(\beta)$ = apparent (orientation-dependent) frequency of the jth fracture set on scanline Beta.

A.1. Development of Eq. (11) for a single fracture set j

Multiplying the true frequency f_j of fracture set j by the Terzaghi correction factor (Terzaghi, 1965) for scanline Beta, we have:

$$f'_j(\beta) = |\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j| f_j. \tag{A1}$$

Eq. (A1) can be re-written as follows:

$$f'_j(\beta) = |\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j| f_j \times \frac{\sum_{k=1}^{m} L_k(\alpha)}{L(\alpha)} = \frac{|\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j|}{L(\alpha)} \sum_{k=1}^{m} \left(\frac{f_j L_k(\alpha) |\hat{\boldsymbol{\alpha}}_k^s\cdot\hat{\mathbf{n}}_j|}{|\hat{\boldsymbol{\alpha}}_k^s\cdot\hat{\mathbf{n}}_j|} \right), \tag{A2}$$

in which the numerator $f_j L_k(\alpha)|\hat{\boldsymbol{\alpha}}_k^s\cdot\hat{\mathbf{n}}_j|$ of the final term is equal to the expected number, $E(N_{j,k})$, of intersections of set j fractures with the kth segment of the scanline Alpha. Making this substitution, we obtain:

$$f'_j(\beta) = \frac{|\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j|}{L(\alpha)} \sum_{k=1}^{m} \left(\frac{E(N_{j,k})}{|\hat{\boldsymbol{\alpha}}_k^s\cdot\hat{\mathbf{n}}_j|} \right). \tag{A3}$$

Since the frequency $f'_j(\beta)$ is itself an expected value, and expectation is a linear operator, we can replace $E(N_{j,k})$ with its point estimate $N_{j,k}$, for $k = 1,2,\ldots,m$. It is now possible to express Eq. (A3) in terms of a summation over the number (N_j) of set-j fractures intersected, rather than over the m segments of scanline Alpha. This leads to:

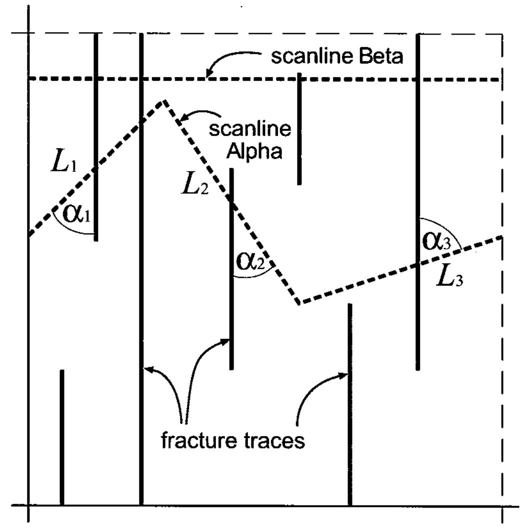

Fig. 8. Sampling a set of parallel fractures. Scanline Alpha has three segments; scanline Beta is a straight line perpendicular to the fractures.

$$f'_j(\beta) = \frac{|\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j|}{L(\alpha)} \sum_{i=1}^{N_j} \left(\frac{1}{|\hat{\boldsymbol{\alpha}}_i^f\cdot\hat{\mathbf{n}}_j|} \right) = \frac{1}{L(\alpha)} \sum_{i=1}^{N_j} \left(\frac{|\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j|}{|\hat{\boldsymbol{\alpha}}_i^f\cdot\hat{\mathbf{n}}_j|} \right), \tag{A4}$$

which is the same as Eq. (11) for a single fracture set j.

A.2. Development of Eq. (11) for multiple fracture sets

Frequencies of individual fracture sets can be added to obtain the total fracture frequency on scanline Beta for the q joint sets, as follows:

$$f'(\beta) = \sum_{j=1}^{q} f'_j(\beta) = \frac{1}{L(\alpha)} \sum_{j=1}^{q} \sum_{i=1}^{N_j} \left(\frac{|\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_j|}{|\hat{\boldsymbol{\alpha}}_i^f\cdot\hat{\mathbf{n}}_j|} \right), \tag{A5}$$

which simplifies to:

$$f'(\beta) = \frac{1}{L(\alpha)} \sum_{i=1}^{N} \left(\frac{|\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}_i|}{|\hat{\boldsymbol{\alpha}}_i^f\cdot\hat{\mathbf{n}}_i|} \right). \tag{A6}$$

Eq. (A6) is the same as Eq. (11).

A.3. Example with a single fracture set

We consider an example based upon the single set of parallel fracture traces shown in Fig. 8. Scanline Alpha is of length L, consists of three segments of lengths L_1, L_2 and L_3, and intersects fractures at four locations. Scanline Beta is to be oriented perpendicular to the fractures, so that $\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{n}}$ is a constant and equal to one. The frequency $f'(\beta)$ of this single set of fractures on scanline Beta is by inspection equal to the true frequency, f. We confirm this result by using the methods presented in this paper. From Eq. (A6):

$$f'(\beta) = \frac{1}{L}\left(\frac{N_1}{\cos\alpha_1} + \frac{N_2}{\cos\alpha_2} + \frac{N_3}{\cos\alpha_3} \right), \tag{A7}$$

where N_1, N_2 and N_3 are the numbers of intersections with segments 1, 2 and 3, respectively. Let $f'(\alpha_1)$, $f'(\alpha_2)$ and $f'(\alpha_3)$ denote the apparent frequencies along the three segments of scanline Alpha, so that $E(N_i) = f'(\alpha_i)L_i$ for $i = 1$, 2 and 3. Then:

$$f'(\beta) = \frac{1}{L}\left(\frac{f'(\alpha_1)L_1}{\cos\alpha_1} + \frac{f'(\alpha_2)L_2}{\cos\alpha_2} + \frac{f'(\alpha_3)L_3}{\cos\alpha_3}\right)$$
$$= \frac{1}{L}\left(\frac{f\cos\alpha_1 L_1}{\cos\alpha_1} + \frac{f\cos\alpha_2 L_2}{\cos\alpha_2} + \frac{f\cos\alpha_3 L_3}{\cos\alpha_3}\right) = f. \quad \text{(A8)}$$

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